Philosophy 324A
Philosophy of Logic
2016

Note Eight

THE CC SPECTRUM AND FORMAL REPRESENTAILITY

Before getting down to business please be aware that by now you should have read or been reading (not just skimming) the first four chapters of Beall and Restall

1. *Rational reconstruction*

We said earlier that the formal representability relation is the driving force behind 20th and 21st century analytic philosophy (minus the ordinary language movement). This can be illustrated by asking what should be an obvious question:

- *If we wanted to implement the rational reconstruction option, how would we go about producing it?*

If we followed Tarski's example in "The concept of truth in formalized languages", we would haul out the CC spectrum. To keep things short, I'll schematize. Let "IC" denote an intuitive concept whose clarification we seek to acquire. Let "A(IC)" denote an *analysis* of the intuitive concept. Let "E(IC)" denote the *explication* of the concept, and "E(A(IC))" denote an explication of the already analyzed concept. Let "RR(IC)" denote a rational reconstruction of the intuitive concept, "RR(A(IC))" a rational reconstruction of the concept as already analyzed, "RR(E(IC))" a rational reconstruction of (E(A(IC))). An RR-target is a concept. An *RR-property* is a property of an RR- targets instantiations. If you are in the rational reconstruction business, you have at least four basic reconstruction targets to consider:

- IC
- A(IC)
- E(IC)
- E(A(IC)))

Other options could be considered, e.g.

- A(E(IC)).

2. *Concept-bending*

As far as I know, no rational reconstructionist in the period under review has made mention of the make-up and complexities of an intuitive concept's reconstruction space.

It is interesting to contrast the philosophical clarification of concepts with a scientist's *managing of data.* Empirical science seeks to discover the underlying organization and predictive capacities of observational data. It rarely, if ever, happens that science takes its "raw" data just as they come. As we saw earlier, two of the basic challenges for empirical science is getting hold of the right data (the *data-collection* problem) and clarifying the data in ways that make them theoretically negotiable; in other words cleaning them up so as to make them graspable by the theory's mathematical instruments (the *data-analysis* problem). Sometimes this data-massaging is done tendentiously, with a view to reshaping data to fit a theory's desired outcomes. When the desired outcome is enshrined in scientifically unjustified theoretical bias, data-analysis becomes what Gerd Gigerenzer scorns as *data-bending.*

For most of its history, concepts rather than empirical observations have been the motivating data for analytic philosophers. Here, too, there is a data-analysis requirement, and here too it carries the risk of *concept-bending*, that is, the risk of arriving at clarifications in which the original concepts are no longer recognizable.

Stipulations are a thing part. When a theorist brings a new and wholly original concept into play, he brings it about by making it up. In a sense, stipulation doesn't belong in the concept-clarification spectrum. To be a perfectly fashioned and genuinely useful thing, a stipulated concept need stand in no relation to an intuitive one, besides not being it. Although true as far as it goes, we should omit the fact that usually a brand new concept hasn't much of a future unless it manages to integrate profitably with established concepts already in use. This was true of Gell-Mann's mad-up concept of quark in 1964, and it was true of Russell's made-up concept of set in the aftermath of the paradox. Had Russell's new concept not worked well with various other mathematical concepts, we might never eventually arrived at the ZF concept, which many of today's set theorists now regard as the *intuitive* concept of set!

3. *Tarski's insight*

Let's say this at the beginning. When I speak of Tarski's insight I don't mean to imply that Tarski knew that he had it or even what it was. It is true that Tarski thought that he was on to something original and important, something no less than the rescue of natural language truth by means of a rigorous theory that its exposure to the Liar not matter. (Although, he wouldn't have said it this way. He really didn't quite know what made his treatment of truth a rescue of it.) Tarski writes clumsily and obscurely. His paper is dead easy to get dead wrong. Perhaps this is largely explained by the project's pioneering character – one can't quite see the forest because of all those trees. Suffice it to say that Tarski seems not to understand what he was doing, any more than Donald Davidson would understand what he himself was doing in the late 1950s and the 60s. (I'll come back to Davidson a bit later in the course.)

Well, then, what *was* Tarski doing and *how* was he doing it?

(1) What Tarski was doing was providing a *rational reconstruction* of one or other of the following five conceptual entities in the target space.

- the intuitive concept of truth.
- the analyzed concept of truth
- the explicated concept of truth
- the analysis of the explicated concept of truth
- the explication of the analyzed concept of truth.

Call these his *RR-targets.* An RR-target is a concept. An *RR-property* is a property of an RR-targets instantiations. If the RR-target were the intuitive concept of logical consequence, the RR-properties would be the properties possessed by the intuitive relation of consequence – properties such as, reflexivity, asymmetry, transivity, truth-preservation, and so on.

Similarly for stipulated concepts. They too have instantiations which in turn properties of interest. Call these S-properties. Consider, for example, the stipulated concepts of the model theory of classical logic. Take the stipulated concept of logical truth, that is, the property of having a model in all model-theoretic interpretations of logic's formal language. Logical truth would be an S-property, as would properties of *its* instantiations in turn. We are now in a position to answer question (2).

(2) Tarski achieved his rational reconstruction of truth by way of formal representability relations mapping S-properties of model theory to RR-properties of natural language truth.

This doesn't mean that the rationally reconstructed concept instantiates the properties of the stipulated concept. The idea, rather, is that the stipulated concept's *theory* would serve as a *template* for a good theory of the rationally reconstructed concept, made so by the structural similarities between them. The RR of truth doesn't tell us what the right theory of RR-truth actually is. It tells us only what such a theory should be like.

4. *Canonical notation again*

Let's go back to ZF for a moment. Why has ZF flourished in modern mathematics, given that the ZF concept of set was a made-up one? The answer lies in part in its usefulness in everyday high quality mathematics. For this to have been possible, it cannot have been true that the ZF concept and the intuitive concept are wholly alienated from one another, in spite of the fact that, apart from sharing the name, they share no other property. What would make *that* possible? Evidently, it would have had something to do with some of the structural similarities between old sets and the brand new ones, and would also have had something to do with the comparative ease with which purely stipulated ideas can make fruitful alliances with those already in established and long-lived play. Mathematics owes much of its creative advancement to alliances between the old and the far-flung new. With all the advantages of the Monday morning quarter-back, it is easy for us to see that what happened so fruitfully in modern set theory resulted from putting the vocabulary of old set theory in *canonical notation,* in which the idioms of the new are inserted into the language of the old as neologisms of the old. My surmise is that this wouldn't have worked but for the *formal representation relations* that map old sets to new ones.

The same, we might think, is also true of Tarski. Tarski's theory of truth is a theory in canonical notation, under the influence of deductively tight formal representation relations. This is clearly one point of similarity with ZF set theory. I see a good deal less similarity between the routine value of making things up in mathematics, and making things up in truth theory. Of what value to other productive engines of enquiry would a theory of truth in canonical notation plausibly be? I suppose that at least part of the answer would lie in its contributions to the rest of philosophy, most notably to semantic treatments of scientific theories in the manner of Patrick Suppes (1922-2014).[1]

My one hesitation is that whereas the philosophy of science remains riddled with dissensus, modern arithmetic is comparatively placid and bicker free.

[1] Patrick Suppes, “The role of formal methods in the Philosophy of science”, in P. D. Asquith and H. E. Kyburg, editors, *Current Research in the Philosophy of Science*, pages 16-27, East Lansing: Philosophy of Science Association, 1979. Patrick Suppes, *Representation and Invariance of Scientific Structures,* Stanford: CSLI Publications, 2002.